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Department: Basic Science

Subject: Engg Math-2 (BT-202)

Unit: 5 (Vector)

Vector's

$$\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$$

$$|a| = \sqrt{a_1^2 + b_1^2 + c_1^2}$$

Dot Product

$$\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$$
$$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$$

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$

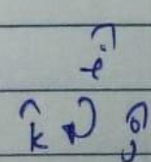
$$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$$

$$\vec{a}\cdot\vec{b} = (a_1\hat{i} + b_1\hat{j} + c_1\hat{k})\cdot(a_2\hat{i} + b_2\hat{j} + c_2\hat{k})$$
$$\vec{a}\cdot\vec{b} = a_1a_2 + b_1b_2 + c_1c_2$$

Cross Procduct

$$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$$

$$\hat{i}\times\hat{j} = \hat{k}$$
$$\hat{j}\times\hat{k} = \hat{i}$$
$$\hat{k}\times\hat{i} = \hat{j}$$



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$\vec{a} \times \vec{b} = (a_1\hat{i} + b_1\hat{j} + c_1\hat{k}) \times (a_2\hat{i} + b_2\hat{j} + c_2\hat{k})$

$a_1b_2\hat{k} + a_1c_2(-\hat{j}) + b_1a_2(-\hat{k}) + b_1c_2\hat{i} + c_1a_2\hat{j} + c_1b_2(-\hat{i})$

$$\vec{a} \times \vec{b} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$\hat{i}(b_1c_2 - c_1b_2)$
$-\hat{j}(a_1c_2 - c_1a_2)$
$\hat{k}(a_1b_2 - b_1a_2)$

Q. if $\vec{a} = t\hat{i} - 2t^2\hat{j} + \hat{k}$
$\vec{b} = \hat{i} - t\hat{j} + 3\hat{k}$

Find out $\vec{a} \cdot \vec{b}$, $\vec{a} \times \vec{b}$, $\frac{d}{dt}\vec{a}$, $\frac{d}{dt}\vec{b}$, $\frac{d^2}{dt^2}\vec{a}$, $\frac{d}{dt}\vec{a} \cdot \frac{d}{dt}\vec{b}$

$\vec{a} \cdot \vec{b} = (t\hat{i} - 2t^2\hat{j} + \hat{k}) \cdot (\hat{i} + t\hat{j} - 3\hat{k})$

$$\boxed{\vec{a} \cdot \vec{b} = t - 2t^3 - 3}$$

$dt = 1\,\text{sec.}$
$\vec{a} \cdot \vec{b} = 1 - 2 - 3 = -4$

$$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ t & -2t^2 & 1 \\ 1 & t & -3 \end{vmatrix}$$

$$\Rightarrow \hat{i}(6t^2 - t) - \hat{j}(-3t - 1) + \hat{k}(t^2 + 2t^2)$$

$$\Rightarrow \boxed{(6t^2 - t)\hat{i} + (3t+1)\hat{j} + 3t^2\hat{k}}$$

$$\frac{d}{dt}\vec{a} = \frac{d}{dt}(t\hat{i} - 2t^2\hat{j} + \hat{k})$$

$$\boxed{\frac{d}{dt}\vec{a} = \hat{i} - 4t\hat{j}} \quad \text{or}$$

$$\left(\frac{d\vec{a}}{dt}\right)_{t=1} = \hat{i} - 4\hat{j}$$

$$\frac{d}{dt}\vec{b} = \frac{d}{dt}(\hat{i} + t\hat{j} + 3\hat{k})$$

$$\frac{d}{dt}\vec{b} = (0 - \hat{j} + 0)$$

$$\boxed{\left(\frac{db}{dt}\right)_{t=1} = \hat{j}}$$

$$\left(\frac{db}{dt}\right)_{t=1} = -4\hat{j}$$

$$\left(\frac{d}{dt}\vec{a} \cdot \frac{d\vec{b}}{dt}\right) = (\hat{i} - 4t\hat{j}) \cdot (\hat{j})$$

$$\boxed{-4t}$$

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\text{Grad } \phi = \vec{\nabla}\phi = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\phi$$

$$\text{Div } \vec{F} = \vec{\nabla}\cdot\vec{a} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(a_1\hat{i} + b_1\hat{j} + c_1\hat{k}\right)$$

$$\text{Curl } \vec{F} = \vec{\nabla}\times\vec{F} \quad \frac{d}{\partial x}a_1 + \frac{\partial}{\partial y}b_1 + \frac{d}{\partial z}c_1$$

Q. if vector are $\vec{r} = a\cos t\hat{i} + a\sin t\hat{j} + at\tan\alpha\hat{k}$

(i) $|r\cdot\dot{r}|$ (ii) $|\dot{r}\times\ddot{r}|$

$$\frac{dr}{dt} = \dot{r}$$

$$\frac{d}{\partial t} = a\cos t\hat{i} + a\sin t\hat{j} + at\tan\alpha\hat{k}$$

$$\frac{d}{dt} = -a\sin t\hat{i} + a\cos t\hat{j} + a\tan\alpha\hat{k}$$

$$\dot{r} = \frac{d}{dt} = (-a\sin t\hat{i} + a\cos t\hat{j} + a\tan\alpha\hat{k})$$

$$\frac{d^2r}{dt^2} = \ddot{r} = \frac{d}{dt}(-a\sin t\hat{i} + a\cos t\hat{j} + a\tan\alpha\hat{k})$$

$$\ddot{r} = -a\cos t\hat{i} - a\sin t\hat{j}$$

$$r \cdot \ddot{r} = (a\cos t\,\hat{i} + a\sin t\,\hat{j} + at\tan\alpha\,\hat{k}) \cdot (-a\sin t\,\hat{i} + a\cos t\,\hat{j} + a\tan\alpha\,\hat{k})$$

$$\Rightarrow \quad -a^2 \sin t \cos t + a^2 \sin t\cos t + a^2 \tan^2\alpha\, t$$

$$\dot{r} \cdot \ddot{r} = \quad a^2 \tan^2\alpha\, t$$

$$|r \cdot \dot{r}| = a^2 \tan^2\alpha\, t$$

$$|\dot{r} \times \ddot{r}| = (-a\sin t\,\hat{i} + a\cos t\,\hat{j} + a\tan\alpha\,\hat{k}) \times (-a\cos t\,\hat{i} - a\sin t\,\hat{j})$$

$$|\dot{r} \times \ddot{r}| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -a\sin t\,\hat{i} & a\cos t\,\hat{j} & a\tan\alpha\,\hat{k} \\ -a\cos t\,\hat{i} & a\sin t\,\hat{j} & 0 \end{vmatrix}$$

$$(a^2\tan\alpha\sin t)\,\hat{i} - (a^2\cos t\tan\alpha)\,\hat{j} + (a^2\sin^2 t + a^2\cos^2 t)\,k$$

$$a^2\tan\alpha\sin t\,\hat{i} - a^2\cos t\tan\alpha\,\hat{j} + a^2 k$$

$$|\dot{r} \times \ddot{r}| = \sqrt{a^4\tan\alpha\sin t\,\hat{i} + a^4\tan^2\alpha\cos^2 t + a^4}$$

$$|\dot{r} \times \ddot{r}| = \sqrt{\tan^2\alpha\,(\sin^2 t + \cos^2 t) + 1}$$

$$|\dot{r} \times \ddot{r}| = a^2\sqrt{\tan^2\alpha\,(1) + 1}$$

$$= a^2\sqrt{\tan^2\alpha + 1}$$

$$a^2 \sec^2\alpha$$

Ans

Q. Find out velocity and acceleration of the curve $x = 4\cos t$ $y = 4\sin t$ and $z = 6t$ at the time $t = 0$ $t = \pi/2$

Given $x = 4\cos t$, $y = 4\sin t$, $z = 6t$

we know that

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{r} = 4\cos t\,\hat{i} + 4\sin t\,\hat{j} + 6t\,\hat{k}$

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{a} = \frac{d}{dt}v = \frac{d^2\vec{r}}{dt^2}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(-4\sin t\,\hat{i} + 4\cos t\,\hat{j} + 6\hat{k})$$

$$\vec{a} = \frac{d}{dt}v = \frac{d^2\vec{r}}{dt^2} = \frac{d}{dt}(-4\sin t\,\hat{i} + 4\cos t\,\hat{j} + 6\hat{k})$$

$$= (-4\cos t\,\hat{i} - 4\sin t\,\hat{j})$$

$t = 0$

$$\vec{v} = \left(\frac{d\vec{r}}{dt}\right)_{t=0} = -4\sin 0\,\hat{i} + 4\cos 0\,\hat{j} + 6\hat{k}$$

$$= 4\hat{j} + 6\hat{k}$$

$$|\vec{v}| = \sqrt{-4^2 + 6^2} = \sqrt{52}$$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = -4\cos 0\,\hat{i} - 4\sin 0\,\hat{j}$$

$$= -4\hat{i} = 4 \quad //$$

Q. if a, b is a cofficient vector ω is a constant r is a vector fuction of the scaler veriable.

$$\vec{r} = a\cos\omega t\,\hat{i} + a\sin\omega t\,\hat{j}$$

$$\frac{\partial\vec{r}}{\partial t} = \frac{\partial}{\partial t}(a\cos\omega t\,\hat{i} + a\sin\omega t\,\hat{j})$$

$$= -a\sin\omega t\,\hat{i} + a\cos\omega t\,\hat{j}$$

$$\frac{d^2\vec{r}}{dt^2} = \frac{d}{dt}(-a\sin\omega t\,\hat{i} + a\cos\omega t\,\hat{j})$$

$$\vec{r} = -a\cos\omega t\,\hat{i} - a\sin\omega t\,\hat{j}$$

$$\frac{d^2 r}{dt^2} = -\omega^2(a\cos\omega t\,\hat{i} + a\sin\omega t\,\hat{j})$$

$$\frac{d^2 r}{dt^2} = -\omega^2 r$$

$$\boxed{\frac{d^2 r}{dt^2} + \omega^2 r = 0}$$

* $\vec{r} \times \dot{\vec{r}}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a\cos t & a\sin t & 0 \\ -a\sin\omega t & a\cos\omega t & 0 \end{vmatrix}$$

$\Rightarrow$ $\hat{i}(a\cos\omega t \times 6t) + 6t(a\sin\omega t) + \hat{j}((4\cos t \times \cos\omega t)$
$- (4\sin t \times a\sin\omega t)$

$\Rightarrow$ ~~$6a\cos\omega t^2 + 6a\sin\omega t^2 + 4\cos^2\omega t - 4a\sin^2 t$~~

$\Rightarrow$ $\hat{i}(0) - \hat{j}(0) + k(a^2\omega\cos^2\omega t + a^2\omega\sin^2\omega t)$

$\Rightarrow$ ~~$k(a^2\omega(\cos^2\omega t + \sin^2\omega t)$~~

$\Rightarrow$ $a^2\omega\hat{k}$.

Q. Find out gradnet of function

$x^3+y^3+z^3-x-y-z$ at a Point $(1,1,-1)$

$$\text{grad } \phi = \vec{\nabla}\phi = \left(\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right)\phi$$

$$= \frac{d}{dx}\phi\hat{i} + \frac{d}{dy}\phi\hat{j} + \frac{d}{dz}\phi\hat{k}$$

$$\frac{d}{dx}(x^3+y^3+z^3-x-y-z)\hat{i} + \frac{d}{dy}(x^3+y^3+z^3-x-y-z)\hat{j} +$$
$$\frac{d}{dz}(x^3+y^3+z^3-x+y-z)\hat{k}$$

$$\text{grad} = (3x^2-1)\hat{i} + (3y^2-1)\hat{j} + (3z^2-1)\hat{k}$$

$$\text{grad}(1,1,-1) = 2\hat{i} + 2\hat{j} + 2\hat{k})$$

Normal vector $\hat{n} = \dfrac{\text{gra}l\ \phi}{|\text{gra}l\ \phi|}$

$$\hat{n} = \frac{2\hat{i}+2\hat{j}+2\hat{k}}{\sqrt{2^2+2^2+2^2}}$$

$$\frac{2\hat{i}+2\hat{j}+2\hat{k}}{2\sqrt{3}}$$

$$\hat{n} = \frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$$

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Q. Find out the normal vector of given function $3x^2y - y^3z^3$ at a Point $(1,1,1)$

$$\text{grad } \phi = \vec{\nabla} \cdot \phi = \left(\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right)\phi$$

$$= \frac{d}{dx}\phi\hat{i} + \frac{d}{dy}\phi\hat{j} + \frac{d}{dz}\phi\hat{k}$$

$$\frac{d}{dx}(3x^2y - y^3z^2)\hat{i} + \frac{d}{dy}(3x^2y - y^3z^2)\hat{j} + \frac{d}{dz}(3x^2y - y^3z^2)$$

$$(6x)\hat{i} + (1-3y^2)\hat{j} + (2z)k$$

$$\text{grad } (1,1,1) \Rightarrow (6(1)\hat{i} + (1-3(1)^2\hat{j} + (2(1))k$$

$$\Rightarrow 6\hat{i} + 2\hat{j} + 2\hat{k}$$

Q. if $u = x+y+z$, $v = x^2+y^2+z^2$ and $w = yz+zx+xy$ then find out grad u, grad v, grad w

$$u = x+y+z$$

$$\text{grad } u = \frac{d}{dx}u\hat{i} + \frac{d}{dy}v\hat{j} + \frac{d}{dz}u\hat{k}$$

$$u = \frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}k \quad \text{— (A)}$$

$$\frac{d}{dx}v\hat{i} + \frac{d}{dy}v\hat{j} + \frac{d}{dz}\hat{k}$$

$$\frac{d}{dx}(x^2+y^2+z^2)\hat{i} + \frac{d}{dy}(x^2+y^2+z^2)\hat{j} + \frac{d}{dz}(x^2+y^2+z^2)\hat{k}$$

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$$\Rightarrow \frac{\partial}{\partial x} 2x\hat{i} + \frac{\partial}{\partial y} 2y\hat{j} + \frac{\partial}{\partial z} 2z\hat{k} \quad \text{—— (B)}$$

$$\Rightarrow \frac{\partial}{\partial x} w\hat{i} + \frac{\partial}{\partial y} w\hat{j} + \frac{\partial}{\partial z} w\hat{k}$$

$$\Rightarrow \frac{\partial}{\partial x}(yz+zx+xy)\hat{i} + \frac{\partial}{\partial y}(yz+zx+xy)\hat{j} + \frac{\partial}{\partial z}(yz+zx+xy)\hat{k}$$

$$\Rightarrow \frac{\partial}{\partial x}(z+y)\hat{i} + \frac{\partial}{\partial y}(z+x)\hat{j} + \frac{\partial}{\partial z}(y+x)\hat{k} \quad \text{—— (C)}$$

$\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

div $\vec{a} = \vec{\nabla}\cdot\vec{a}$

$$\left(\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right)\cdot\left(a_1\hat{i} + b_1\hat{j} + c_1\hat{k}\right)$$

div $\vec{a} = \frac{d}{dx}a_1 + \frac{d}{dy}b_1 + \frac{d}{dz}c_1$

curl $\vec{a} = \vec{\nabla} \times \vec{a}$

$$\begin{vmatrix} \hat{i} & \hat{j} & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ a_1 & b_1 & c_1 \end{vmatrix}$$

Q. Find out div and curve of given vector function vector $F = (3x^2+y+z)\hat{i} - (x^2-2xy)\hat{j} + (zy-3xz)$ at a Point. $(1,2,1)$

$\vec{F} = (3x^2+yz)\hat{i} - (x^2-2xy)\hat{j} + (zy-3xz)\hat{k}$

div $\vec{F} = \vec{\nabla}\cdot\vec{F} = \left(\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right)\cdot\left\{(3x^2+yz)\hat{i} - (x^2-2xy)\hat{j} + (zy-3xz)\hat{k}\right\}$

div $\vec{F} = \frac{d}{dx}(3x^2+yz) - \frac{d}{dy}(x^2-2xy) + \frac{d}{dz}(zy-3xz)$

div $\vec{F} = 6x + 2x + (y - 3x)$

div $\vec{F}\,(1,2,1) = 6(1) + 2(1)\,\{2 - 3(1)\} = 7$

$$\text{Curl } \vec{F} = \vec{\nabla} \times \vec{F} \quad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 3x^2+yz & -(x^2-2xy) & (2y-3x^2) \end{vmatrix}$$

$$\hat{i}\left(-(x+y)\,d/dy - \frac{d}{dz}\right) - \hat{j}\left((x+y)\,d/dx - (x+y+1)\,d/dz\right) + \hat{k}\left(d/dx(1) - (x+y+1)\,d/dy\right)$$

$$= -\hat{i} + \hat{j} - \hat{k} \quad \text{An}$$

and also show that vector F with dot product of curl F = 0

$$(-\hat{i}+\hat{j}-\hat{k})\cdot(x+y+1)\hat{i} + \hat{j} - (x+y)\hat{k} \Rightarrow -x-y-1+1+x+y$$

$$= 0$$

Q. if $\vec{F} = (x+y+1)\hat{i} + \hat{j} - (x+y)\hat{k}$ find out curl F and also so that $\vec{F}\cdot \text{curl}\,\vec{F} = 0$

$$\text{curl } \vec{F} = \vec{\nabla} \times \vec{F}$$

$$\vec{F} = (x+y+1)\hat{i} + \hat{j} - (x+y)\hat{k}$$

$$\text{div}\vec{F} = \vec{\nabla}\cdot\vec{F} = \left(\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right)\cdot\left\{(x+y+1)\hat{i} + \hat{j} - (x+y)\hat{k}\right.$$

$$\frac{d}{dx}(x+y+1) - \frac{d}{dy}(\hat{j}) + \frac{d}{dz}(x+y)$$

$$\text{curl } \vec{F} = \vec{\nabla} \times \vec{F} \quad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ x+y+1 & \hat{j} & -(x+y) \end{vmatrix}$$

$\{(-\frac{d}{dy}(x+y) - \frac{d}{dz}(1))\hat{i} - (\frac{d}{dx}(-x+y) - \frac{d}{dz}(x+y+1)) + \hat{k}$

$\{\frac{d}{dx}(1) - \frac{d}{dy}(x+y+1)$

$= -\hat{i} + \hat{j} - \hat{k}$

curl $\vec{F} = 0$

$\hat{i}(x+y+1)\hat{j} - (x+y)\hat{k} \cdot (-\hat{i}+\hat{j}-\hat{k})$

$(x+y+1)+1+(x+y)$

$-\cancel{x}+\cancel{y}-\cancel{1}+\cancel{1}+\cancel{x}+\cancel{y}$

0 //

Q. if $\vec{F} = (\sin y + z)\hat{i} + (x\cos y - z)\hat{j} + (x-y)\hat{k}$ is a erotional vector. It means curl $\vec{F} = 0$

curl $\vec{F} = \vec{\nabla} \times \vec{F}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ \sin y + z & x\cos y - z & (x-y) \end{vmatrix}$$

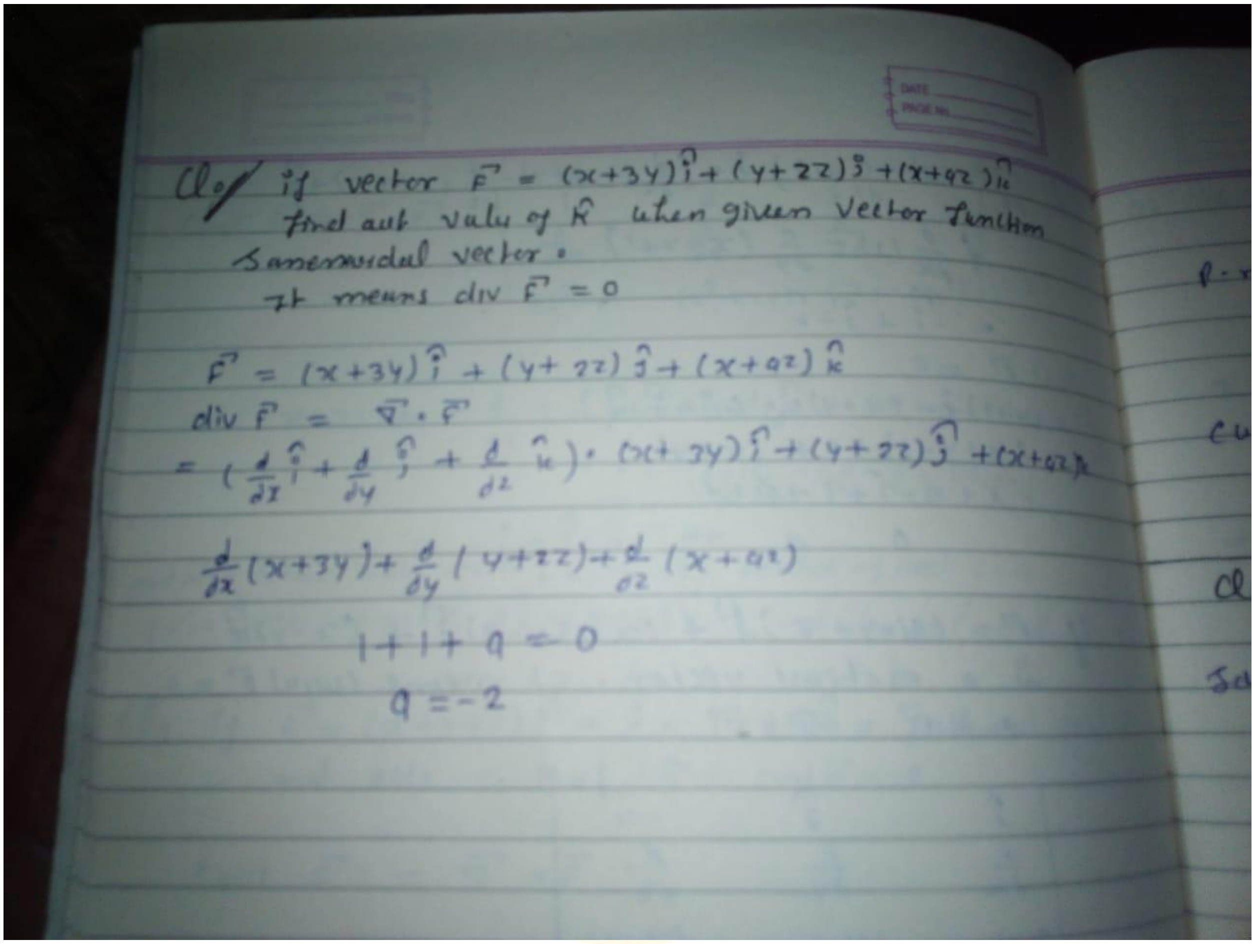
$\hat{i}(\frac{d}{dy}(x-y) - \frac{d}{dz}(x\cos y - z)) - \hat{j}(\frac{d}{dx}(x-y) - \frac{d}{dz}(\sin y + z))$

$\hat{k}(\frac{d}{dx} x\cos y - z - \frac{d}{dy} \sin y + z)$

Parsal Differential

$\hat{i}(-1+1) - \hat{j}(1-1) + \hat{k}(\cos y - \cos y)$

$= 0$



Q. if vector $\vec{F} = (x+3y)\hat{i} + (y+2z)\hat{j} + (x+az)\hat{k}$
find out valu of a when given vector function
Sanenoidal vector.
It means $\operatorname{div} \vec{F} = 0$

$\vec{F} = (x+3y)\hat{i} + (y+2z)\hat{j} + (x+az)\hat{k}$

$\operatorname{div} \vec{F} = \vec{\nabla} \cdot \vec{F}$

$= \left(\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right) \cdot (x+3y)\hat{i} + (y+2z)\hat{j} + (x+az)\hat{k}$

$\frac{d}{dx}(x+3y) + \frac{d}{dy}(y+2z) + \frac{d}{dz}(x+az)$

$1+1+a=0$

$a=-2$

Que Find div $\vec{r}$ and curl $\vec{r}$

Sol $\hat{r} = \frac{\vec{r}}{|r|}$

$$\hat{r} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{3}}$$

$$\text{div } \hat{r} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \left(\frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{3}}\right)$$

$$= \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\text{Curl } \vec{r} = \vec{\nabla} \times \vec{r} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \times \left(\frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{3}}\right)$$

(i) $\text{Div}\ \hat{r} = \left(\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right)\left(\frac{x}{r}\hat{i} + \frac{y}{r}\hat{j} + \frac{z}{r}\hat{k}\right)$

$$= \frac{d}{dx}\frac{x}{r} + \frac{d}{dy}\frac{y}{r} + \frac{d}{dz}\frac{z}{r}$$

$$\frac{r\frac{d}{dx}x - x\frac{dr}{dx}}{r^2} + \frac{r\frac{d}{dy}y - y\frac{dr}{dy}}{r^2} + \frac{r\frac{d}{dz}z - z\frac{dr}{dz}}{r^2}$$

$$= \frac{1}{r^2}\left[\left(r - x\cdot\frac{x}{r}\right) + \left(r - y\cdot\frac{y}{r}\right) + \left(r - z\cdot\frac{z}{r}\right)\right]$$

$$\frac{1}{r^2}\left\{3r - \frac{1}{r}(x^2+y^2+z^2)\right\}$$

$$\frac{1}{r^2}\left\{3r - \frac{1}{r}\cdot r^2\right\}$$

$$\frac{1}{r^2}(3r - r)$$

$$\frac{2r}{r}$$

$$\frac{2}{r} \quad \text{Ans}$$

(ii) $-\text{Curl}\ \hat{r} = \left(\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right)\times\left(\frac{x}{r}\hat{i} + \frac{y}{r}\hat{j} + \frac{z}{r}\hat{k}\right)$

$$\text{Curl}\ \hat{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ \frac{x}{r} & \frac{y}{r} & \frac{z}{r} \end{vmatrix}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{\sqrt{3}} & \frac{y}{\sqrt{3}} & \frac{z}{\sqrt{3}} \end{vmatrix}$$

$$0\hat{i} - (0)\hat{j} + (0)\hat{k} = 0$$ //

Q. find out div of $\vec{r}$ and curl $\vec{r}$

let $r = x\hat{i} + y\hat{j} + z\hat{k}$

$$\text{div } \vec{r} = \nabla \cdot \vec{r} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot (x\hat{i} + y\hat{j} + z\hat{k})$$

$$= \frac{\partial}{\partial x}x + \frac{\partial}{\partial y}y + \frac{\partial}{\partial z}z$$

$\Rightarrow 1+1+1$

$\Rightarrow 3$

Q. find out div $\hat{r}$ and curl $\hat{r}$

We know that

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\hat{r} = \frac{\vec{r}}{|r|} = \frac{r}{r} =$$

$$\hat{r} = \frac{x}{r}\hat{i} + \frac{y}{r}\hat{j} + \frac{z}{r}\hat{k}$$

$$\text{curl } \hat{r} = \hat{i}\left\{\frac{d}{dy}\frac{z}{r} - \frac{d}{dz}\frac{y}{r}\right\} - \hat{j}\left\{\frac{d}{dx}\frac{z}{r} - \frac{d}{dz}\frac{x}{r}\right\} + \hat{k}\left\{\frac{d}{dx}\frac{y}{r} - \frac{d}{dy}\frac{y}{r}\right\}$$

$$\Rightarrow \hat{i}\left\{-\frac{z}{r^2}\frac{dr}{dy} - -\frac{y}{r^2}\frac{dr}{dz}\right\} - \hat{j}\left\{-\frac{z}{r^2}\frac{dr}{dx} - -\frac{x}{r^2}\frac{dr}{dz}\right\} + \hat{k}\left\{-\frac{y}{r^2}\frac{d}{dx}r - -\frac{x}{r^2}\frac{dr}{dy}\right\}$$

$$\Rightarrow \left\{-\frac{z}{r^2}\cdot\frac{y}{r} + \frac{y}{r^2}\cdot\frac{z}{r}\right\} - \hat{j}\left\{-\frac{z}{r^2}\frac{x}{r} + \frac{x}{r^2}\cdot\frac{z}{r}\right\} + \hat{k}\left\{-\frac{y}{r^2}\cdot\frac{x}{r} + \frac{x}{r^2}\frac{y}{r}\right\}$$

$$\hat{i}\left\{\frac{yz}{r^3} - \frac{yz}{r^3}\right\} - \hat{j}\left\{\frac{zx}{r^3} - \frac{zx}{r^3}\right\} + \hat{k}\left\{\frac{xy}{r^3} - \frac{xy}{r^3}\right\}$$

$$\hat{i}(0) - \hat{j}(0) + \hat{k}(0)$$

0 Ans

Q. Find div of $\hat{r}$ & curl of $\hat{r}$

$$\text{div } \hat{r} = \text{div } \frac{\vec{r}}{|r|} = \text{div } \frac{\vec{r}}{r}$$

$$\text{div } \frac{\vec{r}}{r} = \text{div } \frac{(x\hat{i} + y\hat{j} + z\hat{k})}{r}$$

$$= \nabla\left(\frac{x}{r}\hat{i} + \frac{y}{r}\hat{j} + \frac{z}{r}\hat{k}\right)$$

$$= \left(\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right)\cdot\left(\frac{x}{r}\hat{i} + \frac{y}{r}\hat{j} + \frac{z}{r}\hat{k}\right)$$

$$= \frac{d}{dx}\frac{x}{r} + \frac{d}{dy}\frac{y}{r} + \frac{d}{dz}\frac{z}{r}$$

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$$\operatorname{div} \vec{r} = \frac{r\, d/dx - x\, dr/dr}{r^2} + \frac{r\frac{dy}{dy} - y\, dr/dy}{r^2} + \frac{r\frac{dz}{dz} - z\frac{dr}{dz}}{r^2}$$

$$\frac{1}{r^2}\left\{ r - x \cdot \frac{x}{r} \right) + r - y \cdot \frac{y}{r}) + (r - z \cdot \frac{z}{r}) \right\}$$

$$\frac{1}{r^2}\left\{ 3r - \frac{1}{r}(x^2+y^2+z^2) \right)$$

$$\frac{1}{r^2}\left\{ 3r - \frac{1}{\cancel{r}} r^{\cancel{2}} \right)$$

$$\frac{1}{r^2} \times 2r$$

$$\frac{2}{r} \quad \text{An}$$

$$\text{curl of } \hat{r} = \vec{\nabla} \times \vec{r}$$

$$\left(\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right) \times \left(\frac{x}{r}\hat{i} + \frac{y}{r}\hat{j} + \frac{z}{r}\hat{k}\right)$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d/dx & d/dy & d/dz \\ x/r & y/r & z/r \end{vmatrix}$$

$$\hat{i}\left(\frac{z}{r} d/dy - d/dz\, y/r\right) - \hat{j}\left(z/r\, d/dx - \frac{d}{dz}\frac{x}{r}\right\} + \hat{k}\left(\frac{d}{dx}\frac{y}{r} - d/dy\, \frac{x}{r}\right)$$

$$= \hat{i}\left(x\frac{dy}{dy} - z\frac{dr}{dy}\right)$$

Q. If vector $\vec{F} = \nabla(x^3+y^3+z^3-3xyz)$ as gradient of $(x^3+y^3+z^3-3xyz)$ when show that given function irrotational vector.

$\vec{F} = \nabla(x^3-y^3+z^3-3xyz)$

gradient $= \vec{\nabla}\phi$

$= \frac{d}{dx}\hat{i}+\frac{d}{dy}\hat{j}+\frac{d}{dz}\hat{k}\cdot(x^3+y^3+z^3-3xyz)$

$\frac{d}{dx}(x^3+y^3+z^3-3xyz)\hat{i}+\frac{d}{dy}(x^3+y^3+z^3-3xyz)+\frac{d}{dz}(x^3+y^3+z^3-3xyz)$

$(3x^2-3yz)\hat{i}+(3y^2-3xz)\hat{j}+(3z^2-3xy)\hat{k}$

Curl $= \vec{\nabla}\times\vec{F}$

$= \frac{d}{dx}\hat{i}+\frac{d}{dy}\hat{j}+\frac{d}{dz}\hat{k}\cdot(3x^2-3yz)\hat{i}+(3y^2-3xz)\hat{j}+(3z^2-3xy)\hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d/dx & d/dy & d/dz \\ 3x^2-3yz & 3y^2-3xz & 3z^2-3xy \end{vmatrix}$$

$\hat{i}(-3x-(3x)-\hat{j}(-3y-(-3y))+\hat{k}(-3z-(3z)) = 0$ //

Q. Find out divergian of $r^n\cdot\vec{r}$

$r^n\vec{r}$

$r^n(x\hat{i}+y\hat{j}+z\hat{k})$

$r^n x\hat{i}+r^n y\hat{j}+r^n z\hat{k}$

div $r^n\vec{r} = \vec{\nabla}\cdot r^n\vec{r}$

$= \left(\frac{d}{dx}\hat{i}+\frac{d}{dy}\hat{j}+\frac{d}{dz}\hat{k}\right)\cdot(r^n x\hat{i}+r^n y\hat{j}+r^n z\hat{k})$

$= \frac{d}{dx}r^n x+\frac{d}{dy}r^n y+\frac{d}{dz}r^n z$

$=\left(r^n\frac{dx}{dx}+xnr^{n-1}\frac{dr}{dx}\right)+\left(r^n+ynr^{n-1}\frac{dr}{dy}\right)+\left(r^n+znr^{n-1}\frac{dr}{dz}\right)$

$= 3r^n + xnr^{n-1}\frac{dr}{dx} + ynr^{n-1}\frac{dr}{dy} + znr^{n-1}\frac{dr}{dz}$

$= 3r^n + nr^{n-1}\left(x\frac{dr}{dx} + y\frac{dr}{dy} + z\frac{dr}{dz}\right)$

$= 3r^n + nr^{n-1}\left(x\frac{x}{r} + y\cdot\frac{y}{r} + z\cdot\frac{z}{r}\right)$

$\Rightarrow 3r^n + nr^{n-1}\left[\frac{x^2}{r} + \frac{y^2}{r} + \frac{z^2}{r}\right]$

$\Rightarrow 3r^n + nr^{n-1}\left(\frac{r^2}{r}\right)$

$\Rightarrow 3r^n + nr^{n-1}(r)$

$\Rightarrow 3r^n + (nr^n)$

$\Rightarrow r^n(3+n)$ Ans

Q. (i) Prove that gradient of $r^n = nr^{n-1}\vec{r}$

(ii) Find out gradient of $\frac{1}{r}$

(iii) Gradient of $\log|r|$

Q. Find out derutianl derivation of the function $\phi = yz + zx + xy$ in the direction of $\hat{i} + 2\hat{j} + 2\hat{k}$ at the point 1, 2, 0

- Gradient $= \vec{\nabla}\vec{F}$

$= \frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\,(yz + zx + xy)$

$\frac{d}{dy}(yz + zx + xy)\hat{i} + \frac{d}{dy}(yz + zx + xy)\hat{j} + \frac{d}{dz}(yz + zx + xy)\hat{k}$

$\Rightarrow (z+y)\hat{i} + (z+x)\hat{j} + (y+x)\hat{k}$

$|\vec{v}| = \sqrt{1^2+2^2+2^2} = \sqrt{1+4+4} = \sqrt{9}$

Q. find all directional derivation of xyz at $(1,1,1)$ in the following direction i) i ii) $-j$ iii) $\hat{i}+\hat{j}+k$

Gradient of xyz

$$= \frac{d}{dx} xyz\, \hat{i} + \frac{d}{dy} xyz\, \hat{j} + \frac{d}{dz} xyz\, \hat{k}$$

$$= yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k}$$

(i) i $\sqrt{(1)^2} = 1$

unit vector $= \frac{i}{1} = i$

$(yz\,\hat{i} + xz\,\hat{j} + xy\,k) \cdot \hat{i}$

$= yz$

$(1)(1) \Rightarrow 1$

(ii) $-\hat{j}$

$\sqrt{(-1)^2} = 1$

unit vector $= -j/1 = -\hat{j}$

$(yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k})\cdot(-\hat{j}) \Rightarrow -xz \Rightarrow -(1)(1) = -1$

(iii) $i + j + k$

$\sqrt{(1)^2+(1)^2+(1)^2} = \sqrt{3}$

unit vector $= \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

$$d.d = \left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)\cdot\left(yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k}\right)$$

$$d.d = \frac{1}{\sqrt{3}}(yz + xz + xy),$$

$$\frac{1}{\sqrt{3}}(1+1+1)$$

$$\frac{3}{\sqrt{3}} = \frac{\sqrt{3}\cdot\sqrt{3}}{\sqrt{3}} = \sqrt{3} \quad //$$

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Q. $\int_1^2 \vec{r} \times \frac{\partial \vec{r}}{\partial t}$ d

if $\vec{r} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$

$\frac{d\vec{r}}{dt} \Rightarrow \frac{d}{dt}(5t^2\hat{i} + t\hat{j} - t^3\hat{k})$

$\vec{r} \cdot \frac{d\vec{r}}{dt} = 10t\hat{i} + \hat{j} - 3t^2\hat{k})$

$\vec{r} \times \frac{d\vec{r}}{dt} = (5t^2\hat{i} + t\hat{j} - t^3\hat{k}) \times (10t\hat{i} + \hat{j} + 3t^2\hat{k})$

$$r \times \frac{d\vec{r}}{dt} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5t^2 & t & -t^3 \\ 10t & 1 & -3t^2 \end{vmatrix}$$

$\Rightarrow (-3t^3 + t^3)\hat{i} - \hat{j}(-15t^4 + 10t^3)\hat{j} + (5t^2 - 10t^2)\hat{k}$

$\Rightarrow (-2t^3\hat{i} + 5t^4\hat{j} - 5t^2\hat{k})$

$\int_1^2 \vec{r} \times \frac{\partial \vec{r}}{\partial t}\, dt$

$\int_1^2 (-2t^3\hat{i} + 5t^4\hat{j} - 5t^2\hat{k}$

$= -2\left[\frac{t^4}{4}\right]_1^2 \hat{i} + 5\left[\frac{t^5}{5}\right]_1^2 \hat{j} - 5\left[\frac{t^3}{3}\right]_1^2 \hat{k}$

$$= -2\left(\frac{16}{4} - \frac{1}{4}\right)\hat{i} + 5\left(\frac{32}{4} - \frac{1}{5}\right)\hat{j} - 5\left(\frac{8}{3} - \frac{1}{3}\right)\hat{k}$$

$$= -2 \times \frac{15}{4}\hat{i} + 5 \times \frac{31}{5}\hat{j} - 5 \times \frac{7}{3}\hat{k}$$

$$= -2t^3\hat{i} + 5t^4\hat{j} - 5t^2\hat{k}$$

$$= -\frac{15}{2}\hat{i} + 31\hat{j} - \frac{35}{3}\hat{k}$$ Ans

Q

if $\vec{r} = t\hat{i} + (1-t^2)\hat{j} + (2t^3)\hat{k}$

$$\left[\frac{8}{3}-\frac{128}{7}-\{0-0\}\right]+\left[\frac{y^{4/3+1}}{7/3}\right]_0^8$$

$$\left[\frac{8}{3}-\frac{128}{7}\right]+\frac{3}{7}\left[y^{7/3}\right]_0^8$$

$$\frac{56-384}{21}+\frac{3}{7}\left[(8)^{7/3}-\{0\}\right]$$

$$-\frac{328}{21}+\frac{3}{7}\left[(2^3)^{1/3\times 7}\right.$$

$$-\frac{328}{21}+\frac{3}{7}[128]$$

$$\frac{384}{7}-\frac{328}{21}$$

$$\frac{1152-328}{21}$$

$$\frac{824}{21}$$

Q. Find out Intergation of $\int \vec{F}\cdot dr$ if $\vec{F}=(2x+y)\hat{i}+(3y-x)\hat{j}$ where c is corrve xy plane started from $(0,0)$ to $(2,0)$ And then $(3,2)$

$$\oint \vec{F}\cdot d\vec{r}=?$$

$$\vec{F}=(2x+y)\hat{i}+(3y-x)\hat{j}$$

Formula $\boxed{y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)}$

Q. find out line Intergation $\int \vec{F}\cdot dr$ where $\vec{F} = (x^2-y^2)\hat{i} + (xy)\hat{j}$ where c is closed area $y = x^3$ from (0,0) TO (2,8)

$$\oint \vec{F}\cdot d\vec{r} = ?$$

$$\vec{F} = (x^2-y^2)\hat{i} + xy\hat{j}$$

$y = x^3$ from (0,0) to (2,8)

$$\oint \vec{F}\,dr = \int (x^2-y^2)\hat{i} + xy\hat{j})\cdot(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

$$\int (x^2-y^2)dx + xy\,dy$$

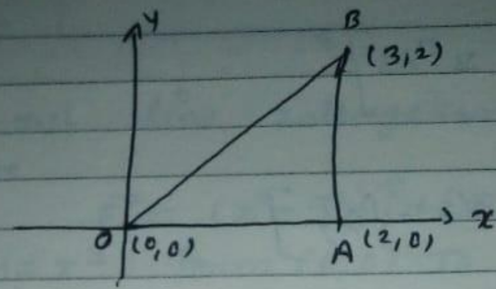
we know that $y = x^3$

$$\int_0^2 (x^2 - x^6)\,dx + \int_0^8 y^{1/3}\,y\,dy$$

$$\left[\frac{x^3}{3} - \frac{x^7}{7}\right]_0^2 + \int_0^8 y^{1/3+1}\,dy$$

$$\left[\frac{x^3}{3} - \frac{x^7}{7}\right]_0^2 + \int_0^8 y^{4/3}\,dy$$

$(0,0)$ to $(2,0)$ and then $(3,2)$



$$\vec{F}\cdot d\vec{r} = \{(2x+y)\hat{i} + (3y-x)\hat{j}\}\cdot\{dx\hat{i} + dy\hat{j} + dz\hat{k}\}$$

$$\vec{F}\cdot d\vec{r} = (2x+y)dx + (3y-x)dy$$

we know that

$$\oint_C \vec{F}\cdot dr + \int_{OA} \vec{F}\cdot dr + \int_{AB} \vec{F}\cdot d\vec{r} + \int_{BO} \vec{F}\cdot d\vec{r}$$

$$\int_{OA} \vec{F}\cdot dr \int_{OA} (2x+y)dx + (3y-x)dy$$

$$\int_{OA} \vec{F}\cdot dr = \int_0^2 (2x+y)dx + \int_0^0 (3y-x)dy$$

$$\int_0^2 2x\,dx + 0$$

$$= 2\left(\frac{x^2}{2}\right)_0^2 = 2\left(\frac{4}{2} - 0\right) = 4$$

$$\int_{AB} \vec{F}\cdot d\vec{r} = \int_{AB} (2x+y)\,dx + (3y-x)\,dy$$

$$y-0 = \frac{2-0}{3-2}(x-2)$$

$$y = 2(x-2)$$

$$x = \frac{y+4}{2}$$

$$\int_2^3 (2x+y)\,dx + \int_0^2 (3y-x)\,dy$$

$$\int_2^3 (2x + 2(x-4))\,dx + \int_0^2 \left(3y - \frac{y+4}{2}\right) dy$$

$$\int_2^3 (4x-4)\,dx + \frac{1}{2}\int_0^2 5y + 4\,dy$$

$$\int_2^3 \cancel{(4x-4)dx + \frac{1}{2} 2\int (5y+4)dy}$$

$$\int_2^3 (4x-4)\,dx + \frac{1}{2}\times 5\int_0^2 y - 4\,dy$$

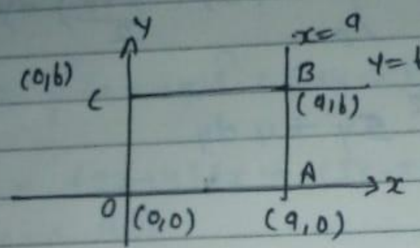
$$\Rightarrow 4\left[\frac{x^2}{2} - x\right]_2^3 + \frac{1}{2}\times 5\left[\frac{y^2}{2} - 4\right.$$

Q. find out line Intrigation $\vec{F}\cdot d\vec{r}$ where $\vec{F} = x^2\hat{i} - xy\hat{j}$ when C is closed area in xy plain bounded by $x=0$, $y=0$, $x=a$, $y=b$.

$$\vec{F}\cdot dr = (x^2\hat{i} - xy\hat{j})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$\oint \vec{F}\cdot d\vec{r} = ?$$

$$\vec{F} = x^2\hat{i} - xy\hat{j}$$



$$\oint \vec{F}\cdot dr + \int_{OA} \vec{F}\cdot dr + \int_{AB} \vec{F}\cdot dr + \int_{BC} \vec{F}\cdot dr + \int_{CO} \vec{F}\cdot dr$$

$$\vec{F} = x^2\hat{i} - xy\hat{j}$$

$$\int_{OA} \vec{F}\cdot dr \int_{OA} x^2dx + xy\,dy$$

$$\int_{OA} \vec{F}\cdot dr \int_0^a x^2dx + \int_0^0 xy\,dy$$

$$\int_{OA} \vec{F}\cdot dr = \int_0^a \left[\frac{x^3}{3}\right]_0^a - \int 0$$

$$= \left[\frac{a^3}{3} - \frac{0}{3}\right]$$

$$\boxed{\int_{OA} \vec{F}\cdot dr = \frac{a^3}{3}}$$

$dy\hat{j} + dz\hat{k})$

$$\int_{AB} \vec{F}\cdot dr \int_{AB} x^2 dx - xy\,dy$$

$$\Rightarrow \int_a^a x^2 dx - \int_0^b xy\,dy \qquad \{x = a\}$$

$$\int_{AB} \vec{F}\cdot dr = \left[\frac{x^3}{3}\right]_a^a - \int_0^b ay\,dy$$

$$\int_{AB} \vec{F}\cdot dr = \left[\frac{x^3}{3}\right]_a^a - a\left[\frac{y^2}{2}\right]_0^b$$

$$= \left[\frac{a^3}{3} - \frac{a^3}{3}\right] - a\left[\frac{b^2}{2} - \frac{0}{2}\right]$$

$$\Rightarrow \boxed{0 - \frac{ab^2}{2}}$$

$$\int_{AB} \vec{F}\cdot dr = -\frac{ab^2}{2}$$

$$\int_{BC} \vec{F}\cdot dr = \int_a^0 x^2 dx - \int_b^b xy\,dy \qquad x = a$$

$$= \left[\frac{x^3}{3}\right]_a^0 - \int_b^b ay\,dy$$

$$\left[\frac{x^3}{3}\right]_a^0 - a\left[\frac{ay^2}{2}\right]$$

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$$= \left[\frac{a^3}{3} - \frac{0}{3}\right] - [0]$$

$$\boxed{\int_{BC} \vec{F} \cdot dr = -a^3/3}$$

$$\oint_{CO} \vec{F} \cdot dr = \int_0^0 x^2 dx - \int_b^0 xy \, dy \qquad \{x = a$$

$$0 - \int_b^0 0(y) \, dy$$

$$\boxed{\oint \vec{F} \cdot dr = 0}$$

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Surface Intrigation

$$\iint \vec{F}\cdot\hat{n}\,ds = \iint \vec{F}\cdot\hat{n}\,\frac{dx\,dy}{|\hat{n}\cdot\hat{k}|} = \iint \vec{F}\cdot\vec{n}\,\frac{dy\,dz}{|\hat{n}\cdot\hat{i}|} = \iint \vec{F}\cdot\vec{n}\,\frac{dx\,dz}{|n\cdot\hat{j}|}$$

Q. find out surface intrization if $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ where is the surface of space $x^2+y^2+z^2-1=0$ in a Ist octen

$$\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$$

$$\phi = x^2+y^2+z^2-1$$

$$\text{grad}\,\phi = \vec{\nabla}\phi = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\phi$$

$$= \frac{\partial}{\partial x}\phi\hat{i} + \frac{\partial}{\partial y}\phi\hat{j} + \frac{\partial}{\partial z}\phi\hat{k}$$

$$\text{grad}\,\phi = \frac{\partial}{\partial x}(x^2+y^2+z^2)\hat{i} + \frac{\partial}{\partial y}(x^2+y^2+z^2)\hat{j} + \frac{\partial}{\partial z}(x^2+y^2+z^2)\hat{k}$$

$$\text{grad}\,\phi = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$

Q. find out line $\int \vec{F} \cdot dr$ of dr $\vec{r} = (3x^2+6y)\hat{i} + 14yz \cdot \hat{j} + 20xz^2 k$ where c is closed area with start at from $(0,0,0)$ to $(1,1,1)$.

$$\oint_c \vec{F} \, dr = \int \{(3x^2+6y)\hat{i} + 14yz\hat{j} + 20xz^2\hat{k}\} \cdot \{dx\hat{i} + dy\hat{j} + dz\hat{k}\}$$

$$\int (3x^2+6y)\,dx + 14yz\,dy + 20xz^2\,dz$$

$$\oint_c \vec{F} \cdot dr = \int_0^1 (3x^2+6y)\,dx + 14\int_0^1 yz\,dy + 20\int_0^1 xz^2\,dz$$

Formula $\quad \frac{x-0}{1-0} = \frac{y-0}{1-0} = \frac{z-0}{1-0} = t$

$$x = y = z = t$$

$x = t \quad y = t \quad z = t$

$dx = dt \quad dy = dt \quad dz = dt$

$$\oint \vec{F} \cdot dr = \int_0^1 (3t^2+6t)\,dt + 14\int_0^1 t^2\,dt + 20\int_0^1 t^3\,d$$

$$\oint \vec{F} \cdot dr =$$

$|grad(\phi)| = \sqrt{(2x)^2+(2y)^2+(2z)^2}$

$\sqrt{4(x^2+y^2+z^2)}$

$\sqrt{4}$

$|grad| = 2$

$\hat{n} = \frac{grad\phi}{|grad\phi|} = \frac{\cancel{2}(x\hat{i}+y\hat{j}+z\hat{k})}{\cancel{2}}$

$\hat{n} = x\hat{i}+y\hat{j}+z\hat{k}$

Given $x^2+y^2+z^2-1=0$

$\hat{j}+$

$x^2+y^2+z^2=1$

$z = \pm\sqrt{1-x^2-y^2}$

$y = \pm\sqrt{1-x^2}$

$x = \pm 1$

$x = 0$ to 1

$y = 0$ to $\sqrt{1-x^2}$

$\hat{n}\cdot\hat{k} = (x\hat{i}+y\hat{j}+z\hat{k})\cdot\hat{k}$

$\hat{n}\cdot\hat{k} = z$

$|\hat{n}\cdot\hat{k}| = z$

$$\iint \vec{F}\cdot\hat{n}\,ds = \iint \vec{F}\cdot\hat{n}\,\frac{dx\,dy}{|\hat{n}\cdot\hat{k}|}$$

$$\iint \left\{ (yz\hat{i} + zx\hat{j} + xy\hat{k})\cdot(x\hat{i} + y\hat{j} + z\hat{k}) \right\} \frac{dx\,dy}{z}$$

$$\iint (xyz + xyz + xyz)\frac{dx\,dy}{z}$$

$$\int_{x=0}^{1}\int_{y=0}^{\sqrt{1-x^2}} 3xy\,dx\,dy$$

$$\int_{x=0}^{1}\left\{\int_{0}^{\sqrt{1-x^2}} y\,dy\right\} 3x\,dx$$

$$\int_{x=0}^{1}\left[\frac{y^2}{2}\right]_{0}^{\sqrt{1-x^2}} 3x\,dx$$

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Q. find out surface $\int \vec{F} \cdot dr$ if
$\vec{F} = y\hat{i} + 2x\hat{j} + z\hat{k}$ and S is a surface of plane $2x + y = 6$ and $z = 4$

$$\iint \vec{F} \cdot \hat{n}\, ds = \iint \vec{F} \cdot \hat{n} \frac{dx\, dz}{|\hat{n} \cdot \hat{j}|}$$

$2x + y = 6$

$y = 6 - 2x$

Let $x = 3$

$x \to 0$ to 3

$z \to 0$ to 4

$\phi = 2x + y - 6$

$$\text{grad}\, \phi = \nabla \phi = \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) \phi$$

$$= \frac{\partial}{\partial x} \phi \hat{i} + \frac{\partial}{\partial y} \phi \hat{j} + \frac{\partial}{\partial z} \phi \hat{k}$$

$$\hat{n}\cdot\hat{k} = \left(\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right)(\hat{k}) \Rightarrow \frac{6}{7}$$

$$\iint \vec{F}\cdot\vec{n}\, ds = \int_{x=0}^{2}\int_{y=0}^{\frac{4-2x}{3}} (18z\hat{i} - 12\hat{j} + 3y\hat{k})\cdot \left(\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) \frac{dx\,dy}{6/7}$$

$$\int_{x=0}^{2}\int_{y=0}^{\frac{4-2x}{3}} \frac{1}{7}\left(36z - 36 + 18y\right)\frac{7}{6}\, dx\, dy$$

$$\int_{x=0}^{2}\int_{y=0}^{\frac{4-2x}{3}} (3y + 6z - 6)\, dx\, dy$$

$$\int_{x=0}^{2}\int_{y=0}^{\frac{4-2x}{3}} 3y + 6\left(\frac{4-2x-3y}{6}\right) - 6\, dx\, dy$$

$$\int_{x=0}^{2}\int_{0}^{\frac{4-2x}{3}} -2x - 2\, dx\, dy$$

$$\int_0^2 \left(-2xy - 2y\right)_0^{\frac{4-2x}{3}}$$

$$\int_0^2 \left[-2x\left(\frac{4-2x}{3}\right) - 2\left(\frac{4-2x}{3}\right) - 0\right] dx$$

$$-\frac{2}{3}\int_0^2 4x - 2x^2 + 4 - 2x)\, dx$$

$$-\frac{2}{3}\left[\frac{4x^2}{2} - \frac{2x^3}{3} + 4x - \frac{2x^2}{2}\right]_0^2$$

$$-\frac{2}{3}\left[\frac{4(4)}{2} - \frac{2(8)}{3} + 4(2) - \frac{2(4)}{2}\right]$$

$$-\frac{2}{3}\left(8 - \frac{16}{3} + 8 - 4\right) = -\frac{2}{3}\left(12 - \frac{16}{3}\right)$$

$$= -\frac{2}{3}\left(\frac{36-16}{3}\right) = -\frac{2}{3}\left(\frac{20}{3}\right) = -\frac{40}{9}$$

$$\text{grad } \phi = \frac{\partial}{\partial x}(2x+y-6)\hat{i} + \frac{\partial}{\partial y}(2x+y-6)\hat{j} + \frac{\partial}{\partial z}(2x+y-6)\hat{k}$$

$$\text{grad } \phi = 2\hat{i} + \hat{j}$$

$$|\text{grad } \phi| = \sqrt{2^2+1^2} = \sqrt{5}$$

$$\hat{n} = \frac{\text{grad } \phi}{|\text{grad } \phi|} = \frac{2\hat{i}+\hat{j}}{\sqrt{5}}$$

$$\iint \vec{F}\cdot\hat{n} \frac{dx\,dz}{|\hat{n}\cdot\hat{j}|} = \int_{x=0}^{3}\int_{z=0}^{4} (y\hat{i}+2x\hat{j}+2\hat{k})\cdot\left(\frac{2\hat{i}+\hat{j}}{\cancel{\sqrt{5}}}\right)\frac{dx\,dz}{1/\cancel{\sqrt{5}}}$$

$$\int_{x=0}^{3}\int_{z=0}^{4} (2y+2x)\,dx\,dz \qquad \{\text{Put } y = 6-2x\}$$

$$\iint \{2(6-2x)+2x\}\,dx\,dz$$

$$\int_{x=0}^{3}\int_{z=0}^{4} (12-2x)\,dx\,dz$$

$$\iint \vec{F}\cdot\hat{n}\,ds = \int_{x=0}^{3} (12-2x)(z)_0^4\,dx$$

$$= \int_{x=0}^{3} 2(6-x)(4-0)\,dx$$

$$8\int_{x=0}^{3} (6-x)\,dx$$

$$8\left[6x - \frac{x^2}{2}\right]_0^3$$

$$8\left[18 - \frac{9}{2}(0-0)\right]$$

$$8\left(\frac{36-9}{2}\right)$$

$= \cancel{108}\ 108$ Ans /

Q. Find out Surface $\int \vec{F}\cdot n\,ds$ if $\vec{F} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and S is the Part of Plane $2x+3y+6z=4$

$$\iint \vec{F}\,\vec{n}\,ds = \iint \vec{F}\cdot\hat{n}\,\frac{dxdy}{|n\cdot\hat{k}|}$$

$2x+3y+6z=4 \quad [z=0]$

$y=0$ to $y = 4-2x/3$

$x=0$ to 2

$\hat{n}\hat{k} = ?$

$\phi = 2x+3y+6z=4$

Gradient $= \nabla\cdot\phi$

$$= \frac{d}{dx}\phi\hat{i} + \frac{d}{dy}\phi\hat{j} + \frac{d}{dz}\phi\hat{k}$$

$$\frac{d}{dx}(2x+3y+6z)\hat{i} + \frac{d}{dy}(2x+3y+6z)\hat{j} + \frac{d}{dz}(2x+3y$$

$2\hat{i}+3\hat{j}+6\hat{k}$

$$\hat{n} = \frac{\text{gradine } \phi}{|\text{gradient } \phi|} = \frac{2\hat{i}+3\hat{j}+6\hat{k}}{\sqrt{2^2+3^2+6^2}} \Rightarrow \frac{2\hat{i}+3\hat{j}+6\hat{k}}{\sqrt{4+9+16}}$$

$$\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$

$$\iint \vec{F}\cdot\hat{n}\,\frac{dx\,dy}{|\hat{n}\cdot\hat{k}|} = -a\left(\frac{a^3}{3}-0\right)$$

$$= \frac{a^4}{3}$$

$$\iint \vec{F}\cdot\hat{n}\,\frac{dx\,dy}{|\hat{n}\cdot\hat{k}|} = \iint (2xy\hat{i} - yz\hat{j} + x^2\hat{k})\cdot\hat{k}\,\frac{dx\,dy}{|\hat{k}\cdot\hat{k}|}$$

$$= \iint x^2\,dx\,dy$$

$$\left[\frac{x^3}{3}\right]_0^a$$

$$\Rightarrow \frac{a^4}{3}$$

$$\iint_{OACB} \vec{F}\cdot\hat{n}\,\frac{dy\,dz}{|\hat{n}\cdot\hat{i}|} = \iint (2xy\hat{i} - yz\hat{j} + x^2\hat{k})\cdot\hat{i}\,\frac{dy\,dz}{|\hat{i}\cdot\hat{i}|}$$

$$= \int_{y=0}^{a}\int_{z=0}^{a} 2xy\,dy\,dz \qquad \text{put } x=a$$

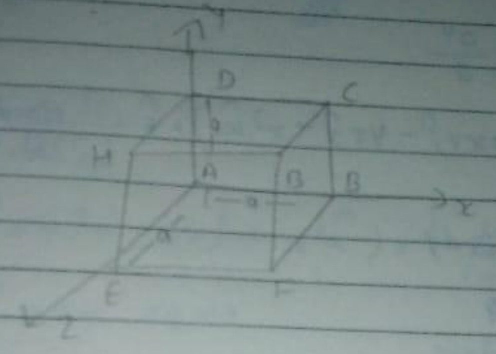
$$\int_{y=0}^{a}\int_{z=0}^{a} 2ay\,dy\,dz$$

$$\int_{y=0}^{a} 2ay\,(z)_0^a\,dy$$

$$\int_{y=0}^{a} 2ay(a-0)\,dy$$

$$2a^2\left(\frac{y^2}{2}\right)_0^a$$

Q. Find out surface Intergation $\vec{F} = 2xy\hat{i} - yz\hat{j} + x^2\hat{k}$ and S is surface of bounded by $x = a,\ y = a,\ z = a$



$$\iint \vec{F}\cdot\hat{n}\,ds = \iint_{ABCD} \vec{F}\cdot\hat{n}\,\frac{dx\,dy}{|\hat{n}\cdot\hat{k}|} + \iint_{HGFE} \vec{F}\cdot\hat{n}\,\frac{dx\,dy}{|\hat{n}\cdot\hat{k}|} + \iint_{GCBF} \vec{F}\cdot\hat{n}\,\frac{dy\,dz}{|\hat{n}\cdot\hat{i}|}$$

$$+ \iint_{HDAE} \vec{F}\cdot\hat{n}\,\frac{dy\,dz}{|\hat{n}\cdot\hat{i}|} + \iint_{DCGH} \vec{F}\cdot\hat{n}\,\frac{dx\,dz}{|\hat{n}\cdot\hat{j}|} + \iint_{ABFE} \vec{F}\cdot\hat{n}\,\frac{dx\,dz}{|\hat{n}\cdot\hat{j}|}$$

$$\iint_{ABCD} \vec{F}\cdot\hat{n}\,\frac{dx\,dy}{|\hat{n}\cdot\hat{k}|} = \iint (2xy\hat{i} + yz\hat{j} + x^2\hat{k})\cdot(-\hat{k})\,\frac{dx\,dy}{|-\hat{k}\cdot\hat{k}|}$$

$$= \int_{x=0}^{a}\int_{y=0}^{a} -x^2\,dx\,dy$$

$$= -\int_{x=0}^{a} x^2 (y)_0^a\,dx$$

$$-\int_{x=0}^{a} x^2 (a-0)\,dx$$

$$-a\left(\frac{x^3}{3}\right)_0^a \Rightarrow -\frac{a^4}{3}$$

$$2a^2\left(\frac{a^2}{2}-0\right)$$

$$= a^4$$

$$\iint_{HDAE} \vec{F}\cdot\hat{n}\ \frac{dy\,dz}{|\hat{n}\cdot\hat{i}|}$$

$$\iint (2xy\hat{i} - yz\hat{j} + x^2\hat{k})\cdot\hat{i}\ \frac{dy\,dz}{|\hat{n}\cdot\hat{i}|}$$

$$\iint_{y=0\ z=0} 2xy\ dy\,dz$$

value of $x=0$

$$= 0$$

$$\iint_{OCGH} \vec{F}\cdot\hat{n}\ \frac{dx\,dz}{|\hat{n}\cdot\hat{j}|} \quad \iint (2xy\hat{i} - yz\hat{j} + x^2\hat{k})\cdot\hat{j}\ \frac{dx\,dz}{|\hat{j}\cdot\hat{j}|}$$

$$\int_{x=0}^{a}\int_{z=0}^{a} -yz\ dx\,dz$$

Put $y=a$

$$\int_{x=0}^{a}\int_{z=0}^{a} -az\ dx\,dz$$

$$\int_{x=0}^{a} -a\left(\frac{z^2}{2}\right)_0^a dx$$

$$\int_{x=0}^{a} a\left(\frac{a^2}{2}-0\right) dx$$

$$-\frac{a^3}{2}\int_{x=0}^{a} dx$$

Volume Intergation

$dv = dx, dy, dz$

Q. Find out volume Intergation of $(2x+y)\,dV$ V is closed regian bounded by $z = 4-x^2$ and its plane $x=0,\ y=0,\ y=2,\ z=0$

$$\iiint (2x+y)\,dv = \iiint 2x+y\ dx\,dy\,dz$$

$z \Rightarrow 0$ to $4-x^2$

$y \Rightarrow 0$ to 2

$x \rightarrow 0$ to 2

$$\int_0^2 \int_0^2 \int_0^{4-x^2} (2x+y)\,dx\,dy\,dz$$

$$\int_{x=0}^{2} \int_{y=0}^{2} \left\{ (2x+y)(z)_0^{4-x^2} \right\} dx\,dy$$

$$\int_{x=0}^{2}\int_{y=0}^{2}\{2x+y(4-x^2-0)\}\,dx\,dy$$

$$\int_{x=0}^{2}\int_{y=0}^{2}\{8x+2x^3+4y-x^2y)\,dx\,dy$$

$$\int_{x=0}^{2}\{\int(8x-2x^3+4y-x^2y)\,dy\}\,dx$$

$$\int_{x=0}^{2}\{8xy-2x^3y+\frac{4y^2}{2}-\frac{x^2y^2}{2}\Big]_0^2\,dx$$

$$\int_0^2\Big[16x-4x^3+\frac{16}{2}-\frac{4x^2}{2}\Big]\,dx$$

$$\int_0^2 \frac{16x^2}{2}-\frac{4x^4}{4}+8x-\frac{2x^3}{3}\Big]_0^2$$

$$\Big[8x^2-x^4+8x-\frac{2x^3}{3}\Big]_0^2$$

$$8(2)^2-(2)^4+8(2)-\frac{2(2)^3}{3}$$

$$32-\cancel{16}+\cancel{16}-\frac{16}{3}$$

$$\frac{96-16}{3}$$

$$\frac{80}{3}$$ //

$$-\frac{a^3}{2}(x)_0^a$$

$$= -\frac{a^4}{2}$$

$$\iint_{ABFE} \vec{F}\cdot\hat{n}\,\frac{dx\,dz}{|\hat{n}\cdot\hat{j}|} = \iint (2xy\,\hat{i} - yz\,\hat{j} + x^2\hat{k})\cdot\hat{j}\,\frac{dx\,dz}{|\hat{j}\cdot\hat{j}|}$$

$$= \iint -yz \qquad y=0$$

$$= 0$$

$$-\frac{a^4}{3} + \frac{a^4}{3} + a^4 + 0 - \frac{a^4}{2} + 0$$

$$= 2a^x - a^x = \frac{a^4}{2}$$

Q. Find out volume $\iiint dv$ where v is closed region bounded by $x+2y+3z=1$

$y \Rightarrow 0$ to

$x \Rightarrow 0$ to 1

$$\iiint (x+2y+3z)\,dx\,dy\,dz$$

$$x = 2y - 3z - 1$$

$$2y = \frac{-3z-1}{2}$$

$$y = -3z-1$$

$$z = -1$$

$$\int_{z=0}^{2y-3z-1} \int_{z=0}^{-3z-1} \int_{z=0}^{(-1)} (x+2y+3z)\,dx\,dy\,dz$$

$$\iint \left(\frac{z^2}{2}\right)_0^{-1} dy\,dz$$

$$\iint \left[\frac{3(-1)^2}{2} - \frac{(0)^2}{2}\right] dy\,dy$$

$$\iint \frac{3}{2}(x+2y+3z)\,dy\,dx$$

$$\frac{3}{2}\int_{x=0}^{2y-3z-1} \int_{y=0}^{-3z-1} (x+2y+3z)\,dy\,dx$$

Gross theorem or Goss Divergen :-

this theorem converd surface intergation into volume intergation. Accarding

$$\iint \vec{F} \cdot \hat{n} \, ds = \iiint div \vec{F} \, dv$$

Stocks theorem :- is theorem converd line intergation into sarface intergation According to this theorem the line intergation of

$$\oint_C \vec{F} \cdot d\vec{r} = \iint curl \vec{F} \cdot \hat{n} \, ds$$

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Q. verify of gass theorem if $\vec{F} = 2xy\hat{i} - yz\hat{j} + x^2\hat{k}$ & where S is sarface of plane baunded by $x = a, \; y = a, \; z = a$

We know that by Gross theorem and Divergen theorem

$$\iint \vec{F} \cdot \hat{n} \, ds = \iiint div F \, dv$$

$$\frac{3}{2}\int^{2y-3z+1}\left(\frac{2y^2}{2}\right)_0^{-3z-1} dx$$

$$= \frac{3}{2}\int^{2y-3z+1}\left(2\left(\frac{-3z-1}{2}\right)^2 - 0\right) dx$$

$$= \frac{3}{2}\int$$

$$L.H.S = \iint \vec{F}\cdot\hat{n}\,ds = \frac{a^4}{2}$$

$$R.H.S = \iiint div\,\vec{F}\,dv$$

$$div\,\vec{F} = \vec{\nabla}\cdot\vec{F} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(2xy\hat{i} - yz\hat{j} + x^2\hat{k}\right)$$

$$\frac{\partial}{\partial x}2xy + \frac{\partial}{\partial y}(-yz) + \frac{\partial}{\partial z}(x^2)$$

$$\boxed{div\,\vec{F} = 2y - z}$$

$$\iiint div\,\vec{F}\,dv = \int_{x=0}^{a}\int_{y=0}^{a}\int_{z=0}^{a}(2y-z)\,dx\,dy\,dz$$

$$= \iint\left\{2y(z)_0^a - \left(\frac{z^2}{2}\right)_0^a\right\}dx\,dy$$

$$= \iint\left(2ay - \frac{a^2}{2}\right)dx\,dy$$

$$= \int\left\{2a\left(\frac{y^2}{2}\right)_0^a - \frac{a^2}{2}(y)_0^a\right\}dx$$

$$= \int\left(a^3 - \frac{a^3}{2}\right)dx = \int\frac{a^3}{2}\,dx$$

$$\frac{a^3}{2}(x)_0^a$$

$$\iiint div\,\vec{F}\,dv = \frac{a^4}{2}$$

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Q. verify of Stocks theorem if $\vec{F} = (x^2+y^2)\hat{i} + 2xy\hat{j}$. where c is closed region bounded by xy plane $x=0$, $y=0$, $x=a$, $y=b$.

we know that by S.T

$$\int \vec{F} \cdot dr = \iint curl\, \vec{F} \cdot \hat{n}\, ds$$

$$L.H.S = \oint \vec{F}\, dr = \frac{-ab^2}{2}$$

Curl $\vec{F} \cdot \hat{n}$ ds $\Rightarrow$

$$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot (x^2\hat{i} - xy\hat{j})$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & xy & 0 \end{vmatrix}$$

$$\left(0 + \frac{\partial}{\partial z}xy\right)\hat{i} - \left(0 - \frac{\partial}{\partial z}x^2\right)\hat{j} + \hat{k}\left(\frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial y}x^2\right)$$

$$0 - 0 + \cancel{x}y - \cancel{x^2}$$

$$\Rightarrow -y\hat{k}$$

$$curl\, \vec{F} \cdot \hat{n} = -y\hat{k} \cdot \hat{k}$$

$$= -y$$

$$\int_{x=0}^{a}\int_{y=0}^{b}(-y)\,dx\,dy$$

$$\int_{x=0}^{a}\left[\frac{-(y)^2}{2}\right]dx$$

$$\int_{x=0}^{a}\left[\frac{-(b^2)}{2}-\frac{0}{2}\right]dx$$

$$\int_{x=0}^{a}\left[\frac{-b^2}{2}\right]dx$$

$$\frac{b^2}{2}\left[-x\right]_{x=0}^{a}$$

$$\frac{-ab^2}{2} \quad //$$

Q. verify of Stocks theorem if $\bar{F} = (x^2+y^2)\hat{i} - 2xy\hat{j}$ where c is closed region bounded by xy plane $x=0$, $y=0$, $x=a$, $y=b$

Q. Div grad r^m

Solution let

$r = \cancel{x^2} + y^2 + z^2$

grad $\vec{\nabla}\,\vec{r}$

$$= \frac{d}{dx}\left(x^2 r^m + \frac{d}{dy} y^2 r^m + \frac{d}{dz} z^2 r^m\right)$$

$$\Rightarrow \frac{d}{dx} x^2 r^m + \frac{d}{dy} y^2 r^m + \frac{\partial}{\partial z} z^2 r^m$$

$$\Rightarrow 2x r^m \cdot r^{m-1} + 2y r^m \cdot r^{m-1} + 2z r^m \cdot r^{m-1}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\text{grad } r^m = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot r^m$$

$$\Rightarrow \frac{\partial}{\partial x} r^m \hat{i} + \frac{\partial}{\partial y} r^m \hat{j} + \frac{\partial}{\partial z} r^m \hat{k}$$

$$\Rightarrow m r^{m-1} \frac{\partial r}{\partial x}\hat{i} + m r^{m-1} \frac{\partial r}{\partial y}\hat{j} + m r^{m-1} \frac{\partial r}{\partial z}\hat{k}$$

$$\Rightarrow m r^{m-1}\left(\frac{r}{x}\right)\hat{i} + m r^{m-1}\left(\frac{y}{r}\right) + m r^{m-1}\left(\frac{z}{r}\right)\hat{k}$$

$$\Rightarrow m r^{m-2} x\hat{i} + m r^{m-2} y\hat{j} + m r^{m-2} z\hat{k}$$

$$\text{div grad } r^m = \vec{\nabla} \cdot \text{grad } r^m$$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \left(m r^{m-2} x\hat{i} + m r^{m-2} y\hat{j} + m r^{m-2} z\hat{k}\right)$$

$$\frac{\partial}{\partial x} m r^{m-2} x + \frac{\partial}{\partial y} m r^{m-2} y + \frac{\partial}{\partial z} m r^{m-2} z$$

$$\Rightarrow 3mr^{m-2} + mr^{m-3}(m-2)\left[\frac{x\,dr}{dx} + \frac{y\,dr}{dy} + \frac{z\,dr}{dz}\right]$$

$$\Rightarrow 3mr^{m-2} + m(m-2)r^{m-3}\left[\frac{x^2}{r} + \frac{y^2}{r} + \frac{z^2}{r}\right]$$

$$\Rightarrow 3mr^{m-2} + m(m-2)r^{m-1}\left(\frac{r^2}{r}\right)$$

$$\Rightarrow 3mr^{m-2} + m(m-2)r^{m-4}\,r^2$$

$$\Rightarrow 3mr^{m-2} + m(m-2)r^{m-2}$$

$$\Rightarrow mr^{m-2}(3 + m-2)$$

$$\Rightarrow mr^{m-2}(m+1)$$

$$\Rightarrow m(m+1)r^{m-2} \quad \text{Ans}$$

Q. Evaluate $\iint \vec{F} \cdot n \, ds$ if $\vec{F} = x\hat{i} - y\hat{j} + z\hat{k}$ and s is surface of $x^2 + y^2 = a^2$ and z is very 0 to 6 by useing gass theorem. in the first actan.

we know that by Gauss Theorem

$$\text{div } \vec{F} = \vec{\nabla} \cdot \vec{F}$$

$$= \left(\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k} \right) \cdot$$

$$x\hat{i} - y\hat{j} + z\hat{k}$$

$$\frac{d}{dx}(x) - \frac{d}{dy}(y) + \frac{d}{dz}(z)$$

$$1 - 1 + 1$$

$$\text{div } \vec{F} = 1$$

$$\int_{x=0}^{a} \int_{y=0}^{\sqrt{a^2 - x^2}} \int_{z=0}^{6} 1 \; dx\,dy\,dz$$

$$\int_{x=0}^{a} \int_{y=0}^{\sqrt{a^2 - x^2}} [6] \; dx\,dy$$

$$6 \int_{0}^{a} \int_{y=0}^{\sqrt{a^2 - x^2}} [y] \; dx$$

$$6 \int_{0}^{a} \sqrt{a^2 - x^2} \,]$$

$$6\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$6\left[\frac{a}{2}\sqrt{a^2-a^2}+\frac{a^2}{2}\sin^{-1}\frac{a}{a}\right]$$

$$6\left(\frac{a^2}{2}\times\frac{\pi}{2}\right)$$

$$\frac{a^2\pi 6}{4} \quad \text{Ans}$$

Q. Evaluate ~~from~~ Prove that

$$\nabla^2 f(\vec{r}) = f''(\vec{r}) + \frac{2}{r}f'(\vec{r})$$

$$\nabla^2 f(\vec{r}) = \vec{\nabla}\vec{\nabla} f(\vec{r})$$

$$\vec{\nabla} f(r) = \left(\frac{d}{dx}\hat{i}+\frac{d}{dy}\hat{j}+\frac{d}{dz}\hat{k}\right)\cdot\left(f(r)\right.$$

$$= \frac{d}{dx}f(r)\hat{i}+\frac{d}{dy}f(r)\hat{j}+\frac{d}{dz}f(r)\hat{k}$$

$$= f'(r)\frac{dr}{dx}\hat{i}+f'(r)\frac{dr}{dy}\hat{j}+f'(r)\frac{dr}{dz}\hat{k}$$

$$\vec{\nabla} f(x) = f'(r)\frac{x}{r}\hat{i}+f'(r)\frac{y}{r}\hat{j}+f'(r)\frac{z}{r}\hat{k}$$

$$\nabla^2 f(x) = \vec{\nabla}(\vec{\nabla}\cdot\vec{F}(x))$$

$$= \frac{d\hat{i}}{dx}+\frac{d\hat{j}}{dy}+\frac{d}{dz}\hat{k}\cdot\left\{f'(x)\frac{x}{r}+f'(r)\frac{y}{r}\hat{i}+f'(r)\frac{z}{r}\hat{k}\right\}$$

$$\frac{d}{dx} f'\left(\frac{x}{r}\right)x = \frac{r\frac{d}{dx} f'(x)x - f'(x)\, x \frac{d}{dx} r}{r^2}$$

$$= \frac{\left[f'(r)\cdot 1 + x f''(r) \frac{dr}{dx} \right] - f'(r)\, x \frac{dr}{dx}}{r^2}$$

$$= \frac{r\left(f'(r) + x f''(r)\, x/r\right] - f''(r)\, x \cdot \frac{x}{r})}{r^2}$$

$$\frac{r f'(r) + x^2 f''(r) - \frac{x^2}{r} f'(x)}{x^2}$$

$$\Rightarrow \frac{f'(r)}{r} + \frac{x^2}{r^2} f''(r) - \frac{x^2}{r^3} f'(r) \quad \text{—— (1)}$$

Similarly

$$\frac{d}{dy} f(r)\frac{y}{r} = \frac{f'(r)}{r} + \frac{y^2}{r^2} f''(r) - \frac{y^2}{r^3} f'(r) \quad \text{—— (2)}$$

Similarly

$$\frac{d}{dz} f'(r)\frac{z}{r} = \frac{f'(r)}{r} + \frac{z^2}{r^2} f''(r) - \frac{z^2}{r^3} f'(r) \quad \text{—— (3)}$$

on adding (1), (2) & (3)

$$\Rightarrow \frac{d}{dx}\left(\frac{x}{r} f'(r) + \frac{d}{dy}\left(\frac{y}{r} f'(r) + \frac{d}{dz}\left(\frac{z}{r} f'(r)\right)\right.\right.$$

$$\Rightarrow \frac{3f'(r)}{r} + \frac{x^2+y^2+z^2}{r^2} f''(r) - \frac{(x^2+y^2+z^2) f'(r)}{r^3}$$

$$\Rightarrow \frac{3f'(r)}{r} + \frac{r^2}{r^2} f''(r) - \frac{r^2}{r^3} f'(r)$$

$$\Rightarrow \frac{3f'(r)}{r} + f''(r) - \frac{f'(r)}{r}$$

$$\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$$